

EJERCICIOS Y PROBLEMAS PROPUESTOS

PARA PRACTICAR

Reglas de derivación

Calcula las derivadas de las siguientes funciones:

1 a) $y = \frac{x^2 - 3}{x^2 + 3}$

b) $y = \sqrt[3]{3x^2}$

a) $y' = \frac{2x(x^2 + 3) - (x^2 - 3)2x}{(x^2 + 3)^2} = \frac{2x^3 + 6x - 2x^3 + 6x}{(x^2 + 3)^2} = \frac{12x}{(x^2 + 3)^2}$

b) $y' = \frac{2}{\sqrt[3]{9x}}$

2 a) $y = \left(\frac{1-x}{1+x}\right)^{2/3}$

b) $y = \frac{2}{x} + \frac{x^2}{2}$

a) $y' = \frac{2}{3} \left(\frac{1-x}{1+x}\right)^{-1/3} \cdot \frac{-1 \cdot (1+x) - (1-x)}{(1+x)^2} = \frac{2}{3} \left(\frac{1+x}{1-x}\right)^{-1/3} \cdot \frac{-1-x-1+x}{(1+x)^2} =$
 $= \frac{2}{3} \frac{-2}{(1-x)^{1/3} \cdot (1+x)^{5/3}} = \frac{-4}{3\sqrt[3]{(1-x)(1+x)^5}}$

b) $y' = 2 \cdot \left(-\frac{1}{x^2}\right) + \frac{1}{2} \cdot 2x = -\frac{2}{x^2} + x$

3 a) $y = \frac{\ln x}{x}$

b) $y = 7e^{-x}$

a) $y' = \frac{(1/x) \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$

b) $y' = -7e^{-x}$

4 a) $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

b) $y = \operatorname{sen} x \cos x$

a) $y' = \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2} = \frac{e^{2x} + e^{-2x} - 2 - e^{2x} - e^{-2x} - 2}{(e^x - e^{-x})^2} = \frac{-4}{(e^x - e^{-x})^2}$

b) $y' = \cos x \cdot \cos x + (-\operatorname{sen} x) \cdot \operatorname{sen} x = \cos^2 x - \operatorname{sen}^2 x = \cos 2x$

18 a) $y = 2^x$

a) $y' = 2^x \cdot \ln 2$

b) $y = \text{arc sen } \frac{x+1}{x-1}$

$$\begin{aligned} \text{b) } y' &= \frac{1}{\sqrt{1 - \left(\frac{x+1}{x-1}\right)^2}} \cdot \frac{(x-1) - (x+1)}{(x-1)^2} = \frac{1}{\sqrt{(x-1)^2 - (x+1)^2}} \cdot \frac{-2}{(x-1)^2} = \\ &= -\frac{2/(x-1)}{\sqrt{(x-1)^2 - (x+1)^2}} = \frac{2}{(x-1)\sqrt{x^2 + 1 - 2x - x^2 - 1 - 2x}} = \\ &= -\frac{2}{(x-1)\sqrt{-4x}} \end{aligned}$$

19 a) $y = 5 \text{ tg}^3(3x^2 + 1)$

a) $y' = 15 \text{ tg}^2(3x^2 + 1) \cdot [1 + \text{tg}^2(3x^2 + 1)] \cdot 6x = 90x [\text{tg}^2(3x^2 + 1) + \text{tg}^4(3x^2 + 1)]$

b) $y = \sqrt{x + \sqrt{x}}$

b) $y' = \frac{1}{2\sqrt{x + \sqrt{x}}} \left(1 + \frac{1}{2\sqrt{x}}\right) = \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}} = \frac{2\sqrt{x} + 1}{4\sqrt{x^2 + x\sqrt{x}}}$

20 a) $y = \sqrt{\text{tg } x^2}$

a) $y' = \frac{1}{2\sqrt{\text{tg } x^2}} (1 + \text{tg}^2 x^2) \cdot 2x = \frac{x(1 + \text{tg}^2 x^2)}{\sqrt{\text{tg } x^2}}$

b) $y = \sqrt[3]{\frac{x-2}{x+2}}$

$$\begin{aligned} \text{b) } y' &= \frac{1}{3} \left(\frac{x-2}{x+2}\right)^{-2/3} \cdot \frac{x+2 - (x-2)}{(x+2)^2} = \frac{1}{3\sqrt[3]{\left(\frac{x-2}{x+2}\right)^2}} \cdot \frac{4}{(x+2)^2} = \\ &= \frac{4}{3 \cdot (x+2)^2 \cdot \frac{\sqrt[3]{(x-2)^2}}{(x+2)^{2/3}}} = \frac{4}{3(x+2)^{4/3} \cdot \sqrt[3]{(x-2)^2}} = \frac{4}{3\sqrt[3]{(x+2)^4 (x-2)^2}} = \\ &= \frac{4}{3(x+2)\sqrt[3]{(x+2)(x-2)^2}} \end{aligned}$$

Otras técnicas de derivación

21 Calcula la derivada de las siguientes funciones, aplicando previamente las propiedades de los logaritmos:

a) $y = \ln \sqrt{\frac{1-x}{1+x}}$

b) $y = \ln(x \text{ tg } x)^2$

c) $y = \ln \left(\frac{\sqrt[3]{x^2 - 1}}{x^2} \right)$

d) $y = \ln(2^x \text{ sen}^2 x)$