Corrección. Números complejos.

1. $\frac{2+xi}{1-xi}\cdot\frac{1+xi}{1+xi}=\frac{2+2xi+xi+x^2i^2}{1+x^2}=\frac{2-x^2}{1+x^2}+\frac{3xi}{1+x^2}$. Para que sea un número real, la parte imaginaria debe ser cero $\frac{3x}{1+x^2}=0\Rightarrow x=0$

2. $3_{60^\circ}=3\left(\cos 60+isen60\right)=\frac{3}{2}+\frac{3\sqrt{3}}{2}i$ $\qquad 3_{120^\circ}=3\left(\cos 120+isen120\right)=-\frac{3}{2}+\frac{3\sqrt{3}}{2}i$

$t=-1+\sqrt{2}i$ $\qquad \left.\begin{array}{l}|t|=\sqrt{(-1)^2+\left(\sqrt{2}\right)^2}=\sqrt{3}\\ \alpha=arctg\frac{\sqrt{2}}{-1}=54^\circ 44'9''\end{array}\right\}\sqrt{3}_{54^\circ 44'9''}$

a) $\left(z+w\right)\cdot\overline{z}=\left(\frac{3}{2}+\frac{3\sqrt{3}}{2}i-\frac{3}{2}+\frac{3\sqrt{3}}{2}i\right)\cdot\left(\frac{3}{2}-\frac{3\sqrt{3}}{2}i\right)=3\sqrt{3}i\left(\frac{3}{2}-\frac{3\sqrt{3}}{2}i\right)=\frac{9\sqrt{3}}{2}i-\frac{9\cdot 3i^2}{2}=\frac{27}{2}+\frac{9\sqrt{3}}{2}i$

b) $\frac{z\cdot w^{10}}{t^2}=\frac{3_{60}\cdot\left(3_{120}\right)^{10}}{\left(\sqrt{3}_{125^\circ 15'52''}\right)^2}=\frac{3_{60}\cdot 3^{10}{}_{10\cdot 120}}{3_{2\cdot 125^\circ 15'52''}}=\frac{3^{11}{}_{60+1200}}{3_{250^\circ 31'44''}}=3^{10}{}_{1009^\circ 28'16''}=3^{10}{}_{289^\circ 28'16''}$

3.-

a) $\sqrt[3]{4-4\sqrt{3}i}$ $\qquad \left.\begin{array}{l}|z|=\sqrt{4^2+4^2\left(\sqrt{3}\right)^2}=\sqrt{16+16\cdot 3}=\sqrt{64}=8\\ \alpha=arctg\frac{-4\sqrt{3}}{4}=arctg\left(-\sqrt{3}\right)=300^\circ\end{array}\right\}8_{300^\circ}$ $\qquad \sqrt[3]{4-4\sqrt{3}i}=\sqrt[3]{8_{300^\circ}}=\left(\sqrt[3]{8}\right)_{\frac{300^\circ}{3}+\frac{360k}{3}}\left\{\begin{array}{l}2_{100^\circ}\\ 2_{220^\circ}\\ 2_{340^\circ}\end{array}\right.$

b) $\left(\frac{3}{2}+\frac{\sqrt{3}}{2}i\right)^6=\left(\sqrt{3}_{30^\circ}\right)^6=3^3{}_{180^\circ}$

4. $\left.\begin{array}{l}z=a+\sqrt{3}i\\ z=2_\alpha\end{array}\right\}$ $\qquad \left.\begin{array}{l}|z|=\sqrt{a^2+3}\\ |z|=2\end{array}\right\}\sqrt{a^2+3}=2\Rightarrow a=\pm 1$ $\qquad \begin{array}{l}Si\ a=1\quad \alpha_1=arctg\frac{\sqrt{3}}{1}=60^\circ\\ Si\ a=-1\quad \alpha_1=arctg\frac{\sqrt{3}}{-1}=120^\circ\end{array}$

5.

a) $\left(5+3i\right)z+\left(4-2i\right)\left(4+2i\right)=\left(3-2i\right)z$

$\left(5+3i\right)z-\left(3-2i\right)z=16+4$

$\left(2+5i\right)z=20$

$z=\frac{20}{2+5i}=\frac{20\cdot\left(2-5i\right)}{\left(2+5i\right)\cdot\left(2-5i\right)}=\frac{40-100i}{4+25}=\frac{40-100i}{29}=$

b) $iz^6 + 3 = 0$ $\qquad \dfrac{-3}{i} \cdot \dfrac{i}{i} = \dfrac{-3i}{-1} = 3i$

$$z = \sqrt[6]{\frac{-3}{i}} = \sqrt[6]{3i} = \sqrt[6]{3_{90}} = \left(\sqrt[6]{3}\right)_{\frac{90}{6}+\frac{360k}{6}} = \begin{cases} z_1 = \sqrt[6]{3}_{15^\circ} \quad z_2 = \sqrt[6]{3}_{75^\circ} \\ z_3 = \sqrt[6]{3}_{135^\circ} \quad z_4 = \sqrt[6]{3}_{195^\circ} \\ z_5 = \sqrt[6]{3}_{255^\circ} \quad z_6 = \sqrt[6]{3}_{315^\circ} \end{cases} =$$

6. Los números complejos que verifican que $\overline{z} + z = 6$, son aquellos que tienen su parte real igualada a 3.